Lecture 36

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7.5 Integration Strategy

Seen a fair number of different integration techniques so far. We will talk about strategies to determine the correct technique to use when faced with an integral:

1) **There is no hard and fast set of rules**. This is just a general set of guidelines that can help us identify the techniques that may work.
2) I personally find it helpful to **rule out certain techniques that you know won't work.** Then you are left with a second list with techniques that might work and work from there.
3) Many integrals can be evaluated in multiple ways and so more than one technique may be use. Sometimes one technique will be significantly easier than others, so don't just stop at the first technique that appears to work. **Take the approach you find easiest.**

1) Simplify the integrand, if possible.

Ex $\int \sqrt{x}(1+\sqrt{x})\,dx$

$$= \int \sqrt{x} + x \, dx = \frac{x^{3/2}}{3/2} + \frac{x^2}{2} + C$$

2) See if a "simple" subsitution will work.

$\int \frac{x}{x^2-1}\,dx$ Method of partial fraction would work.

$u = x^2 - 1$, $du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$

$\int \frac{1}{u}\frac{du}{2} = \ldots$

$$\int \frac{1}{u} \frac{du}{2} = \ldots$$

<u>Ex</u> $\int x\sqrt{x^2-1}\,dx$, $u = x^2 - 1$ ...

3) Identify the type of integral

a) Is the integrand a rat'l function
(Partial fraction).

b) If the integrand is a polynomial times a trig fn, exponential fn, logarithmic, (Integration by parts might work).

$\int x^2 e^x\,dx$ ↗ $u = x^2$
$dv = e^x dx$

<u>I.P</u> $\int u\,dv = uv - \int v\,du$

$u \equiv$ pick a function which simplifies when you take deriv.

$dv \equiv$ pick a function you know how to integrate.

c) If integrand is a product of

sines and cosines, secant & tangent, use trigonometric integral methods.

d) $\sqrt{b^2x^2+a^2}$, $\sqrt{b^2x^2-a^2}$, $\sqrt{a^2-b^2x^2}$ then trig substitution might work.

e) Integrand contains a quadratic in it, then completing squares might put it into a form you know how to deal with.

$$\int \frac{1}{x^2+4x+8}\,dx = \int \frac{1}{\underbrace{(x+2)}^2+4}\,dx$$

$u = x+2$, $du = dx$

$$= \int \frac{1}{u^2+4}\,du = \int \frac{1}{u^2+2^2}\,du$$

$$= \frac{1}{2}\arctan\left(\frac{u}{2}\right)+C$$

5) Might need to use multiple techniques.

6) <u>TRY AGAIN</u>

Ex $\int \frac{\tan x}{\sec^4 x}\, dx$

1

$$= \int \frac{\sin x}{\cos x} \cdot \cos^4 x \, dx$$

$$= \int \sin x \cdot \cos^3 x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$= -\int u^3 \, du = -\frac{u^4}{4} + C = -\frac{\cos^4 x}{4} + C$$

2 $\int \frac{\tan x}{\sec^4 x}\, dx$

$$= \int \frac{\sec x \cdot \tan x}{\sec^5 x}\, dx$$

$u = \sec x$; $du = \sec x . \tan x \, dx$

$$= \int \frac{du}{u^5} = \int u^{-5} du = \frac{u^{-4}}{-4} + C$$

$$= \underline{(\sec x)^{-4}} + C = \underline{\cos^4 x} + C$$

$$= \frac{(\sec x)^{-4}}{-4} + C = \frac{\cos^4 x}{-4} + C$$

<u>Ex</u> $\int \cos(\sqrt{x})\, dx$

$\theta = \sqrt{x}$, $d\theta = \frac{1}{2\sqrt{x}}\, dx \Rightarrow dx = 2\sqrt{x}\, d\theta$

$$\int \cos\theta \cdot 2\sqrt{x}\, d\theta = 2\int \cos\theta \cdot \theta\, d\theta$$

$u = \theta$, $du = d\theta$

$dv = \cos\theta\, d\theta$, $v = \sin\theta$

$$= 2\left[\theta \sin\theta - \int \sin\theta\, d\theta\right]$$

$$= 2\left[\theta \sin\theta + \cos\theta + C\right]$$

$$= 2\left[\sqrt{x}\sin(\sqrt{x}) + \cos\sqrt{x}\right] + C$$

$e^{\sqrt{x}}\, dx$: Use same technique.

Ex $$\int \frac{1}{1+\sin x}\,dx$$

$$= \int \frac{1}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x}\,dx$$

$$= \int \frac{1-\sin x}{\cos^2 x}\,dx$$

$$\frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \tan x \cdot \sec x$$

$$= \int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\,dx$$

$$= \int \sec^2 x - \sec x \tan x\,dx$$

$$= \tan x - \sec x + C$$

Ex $$\int \sqrt{\frac{1-x}{1+x}}\,dx$$

$$= \int \sqrt{1-x \quad 1-x} \quad dx$$

$$= \int \sqrt{\frac{1-x}{1+x} \cdot \frac{1-x}{1-x}} \, dx$$

$$= \int \frac{1-x}{\sqrt{1-x^2}} \, dx$$

$$= \int \frac{1}{\sqrt{1-x^2}} \, dx - \int \frac{x}{\sqrt{1-x^2}} \, dx$$

$u = 1-x^2$

$$= \arcsin x + \sqrt{1-x^2} + C$$

Examples of functions we don't Know how to integrate :

$$\int e^{-x^2} \, dx \quad , \int \cos(x^2) \, dx \quad , \quad \int \frac{\sin(x)}{x} \, dx \ldots$$